Section 4.1: Linear functions and their properties

1 – 10: Find the following:

a) slope

b) y-intercept

c) x-intercept (if any)

d) sketch a graph

e) Determine the interval(s) where the graph is increasing, decreasing or constant.

1) $f(x) = 3x - 6$

1a) $m = 3$

1b) $f(0) = 3(0) - 6$
$= -6$
y-INT $(0,-6)$

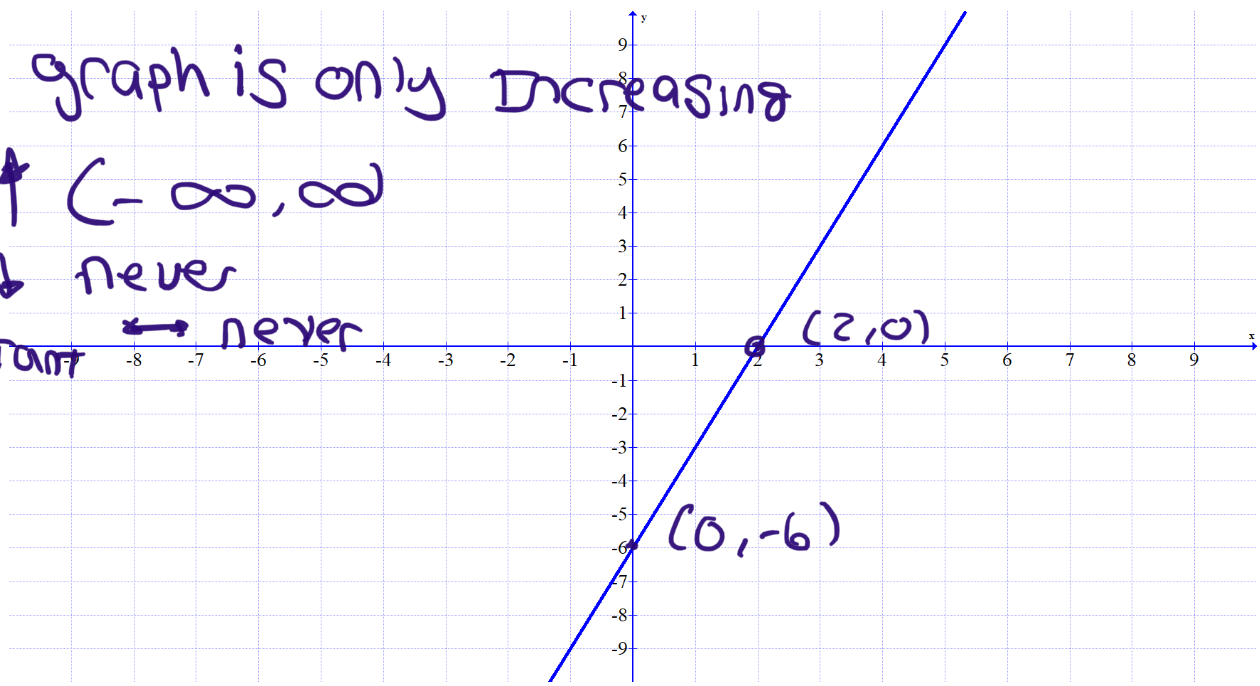
1c) $3x - 6 = 0$
$+6 \;\; +6$
$3x = 6$
$x = 2$
x-INT $(2,0)$

1e) graph is only Increasing

Inc. ↑ $(-\infty, \infty)$

dec. ↓ never

CONSTANT ↔ never



1 – 10: Find the following:

a) slope

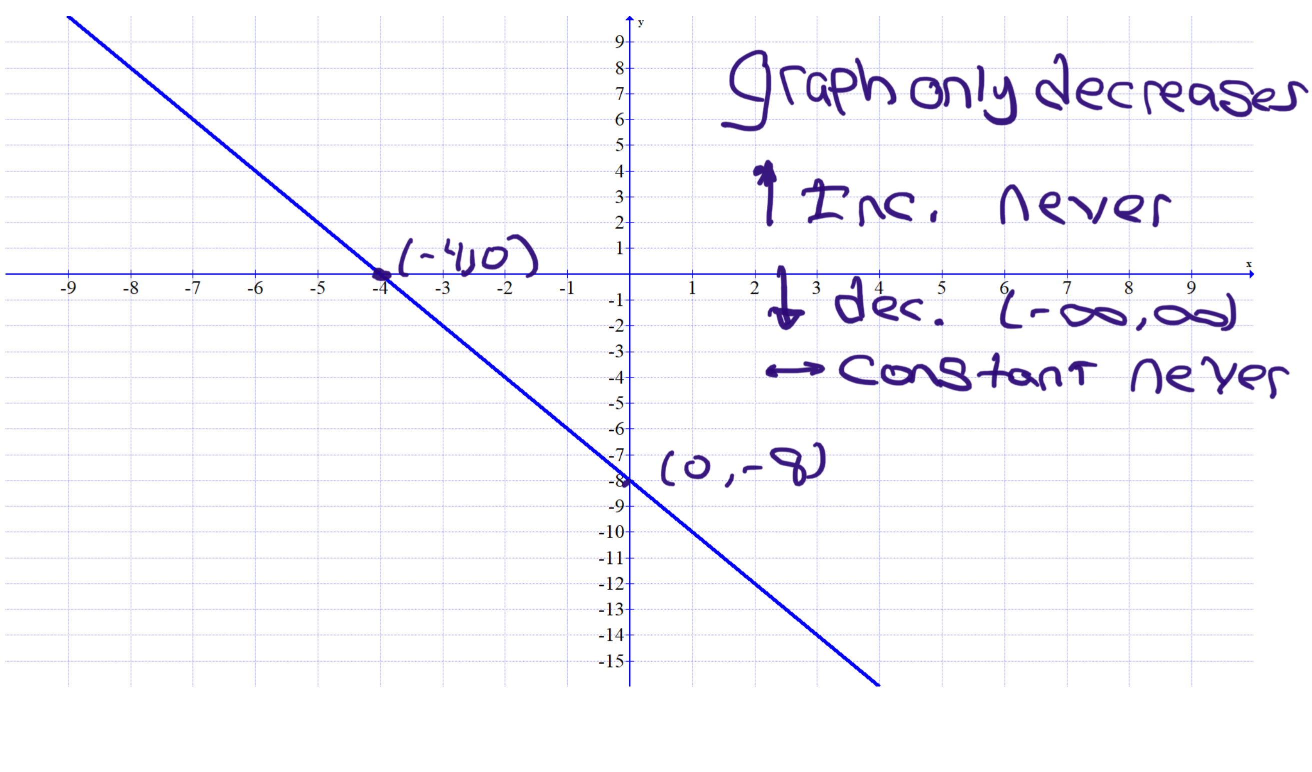
b) y-intercept

c) x-intercept (if any)

d) sketch a graph

e) Determine the interval(s) where the graph is increasing, decreasing or constant.

3) g(x) = -2x – 8



1 – 10: Find the following:

a) slope

b) y-intercept

c) x-intercept (if any)

d) sketch a graph

e) Determine the interval(s) where the graph is increasing, decreasing or constant.

Equation only has a y. will graph as horizontal line through 7 on y-axis

5) f(x) = 7

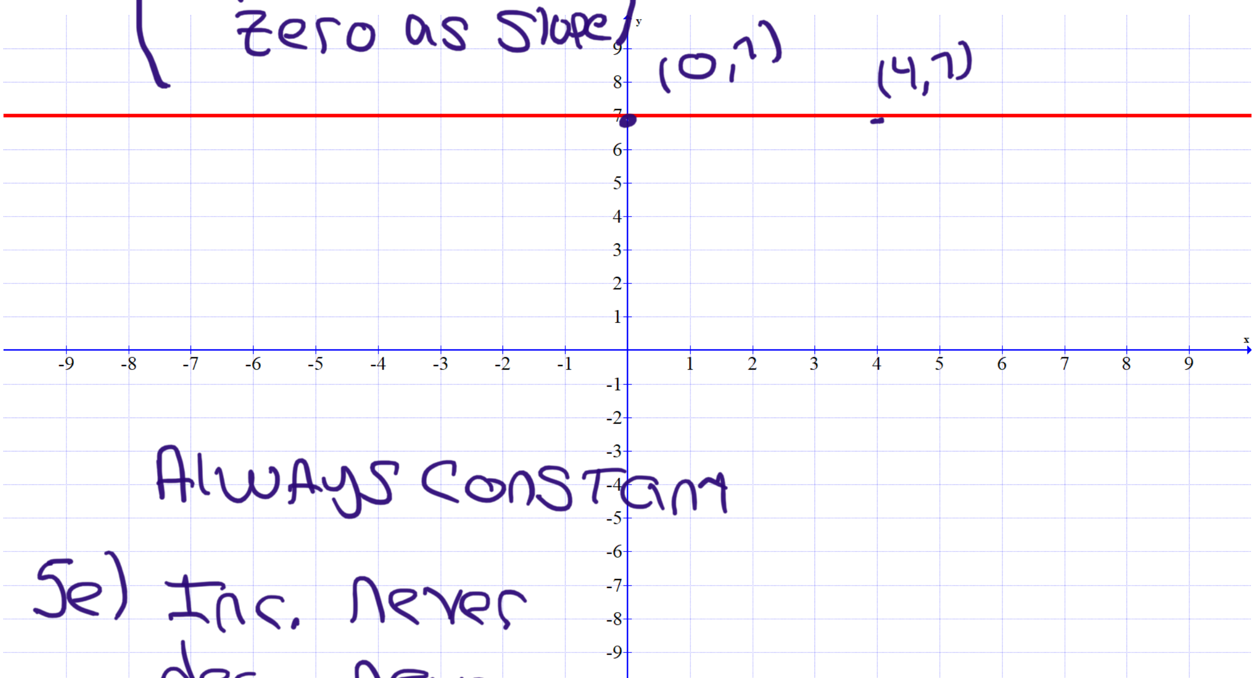
5a) $m = \frac{7-7}{4-0} = \frac{0}{4}$

5b) y-Int (0,7)

5c) none

m = 0

(All horizontal lines have zero as slope)



Always Constant

5e) Inc. never

dec. never

constant $(-\infty, \infty)$

1 – 10: Find the following:

a) slope

b) y-intercept

c) x-intercept (if any)

d) sketch a graph

e) Determine the interval(s) where the graph is increasing, decreasing or constant.

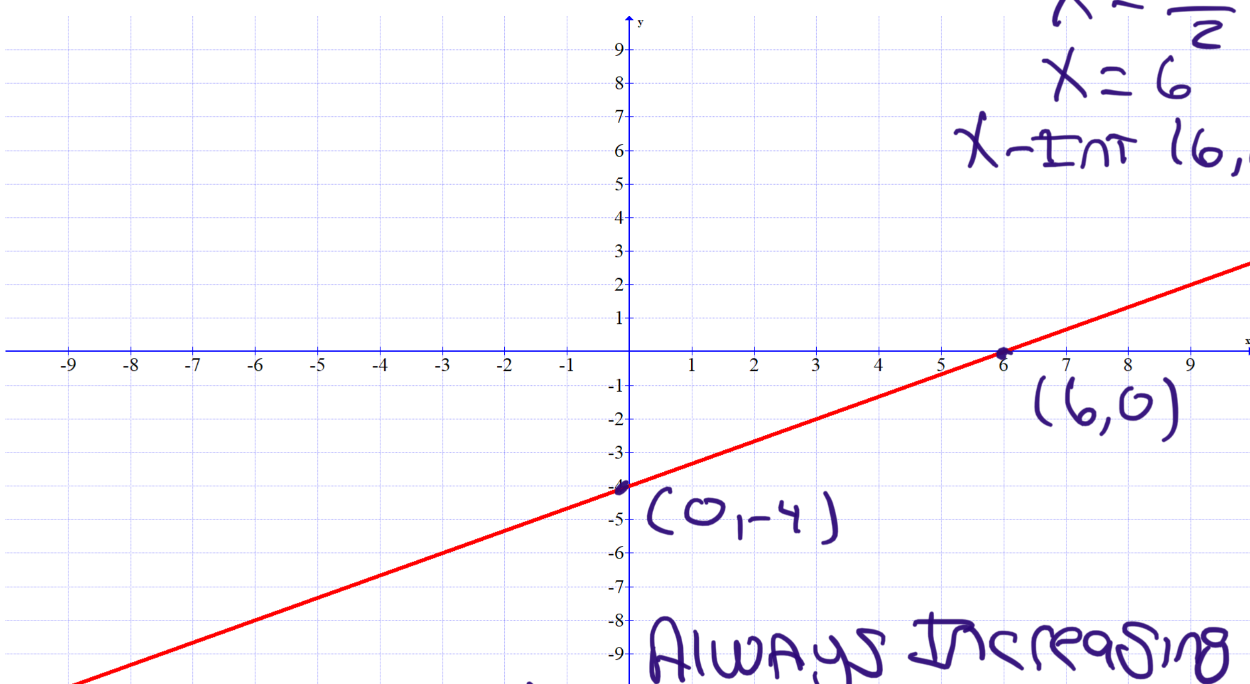
7b) y-Int
(0,-4)

7c)
$\frac{2}{3}x - 4 = 0$
$+4 \quad +4$

$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 4$
$x = \frac{12}{2}$
$x = 6$
x-Int (6,0)

7) $g(x) = \frac{2}{3}x - 4$

7a) $m = \frac{2}{3}$



7e) Inc. $(-\infty, \infty)$
Dec never
constant never

1 – 10: Find the following:

a) slope

b) y-intercept

c) x-intercept (if any)

d) sketch a graph

e) Determine the interval(s) where the graph is increasing, decreasing or constant.

9a) $m = -\frac{1}{4}$ 9b) y-Int (0,2)

9) $f(x) = \frac{-x}{4} + 2$ $= -\frac{1}{4}x + 2$

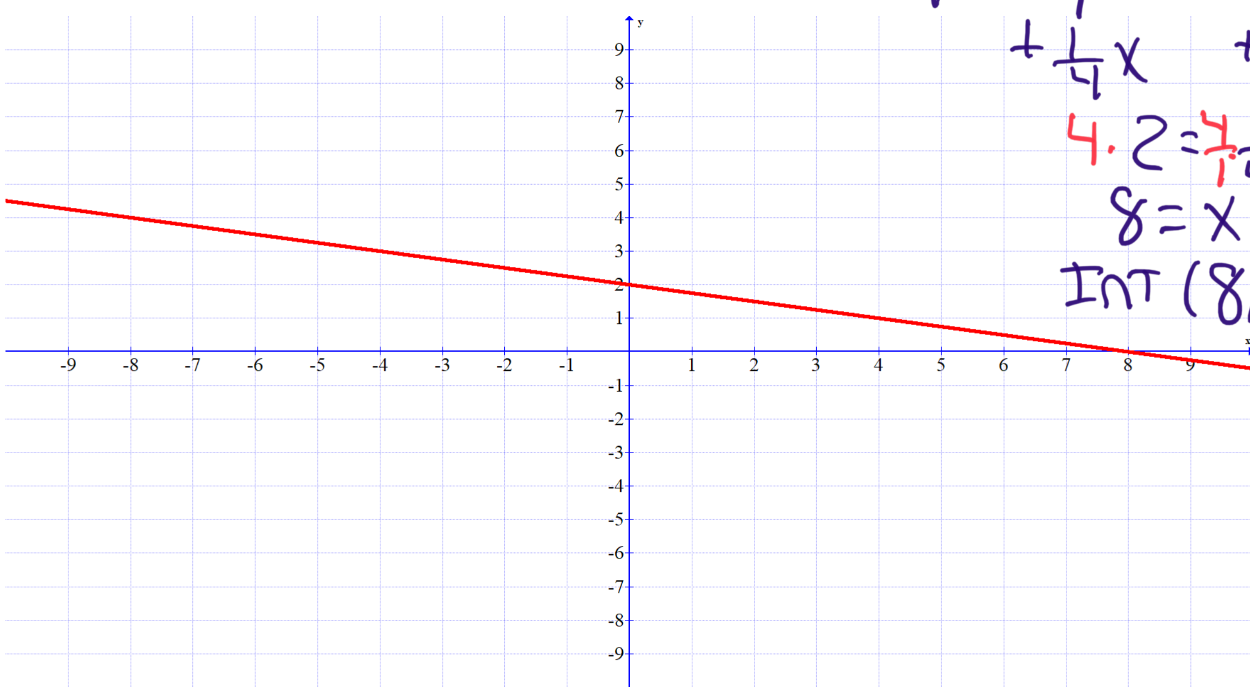
9c) $-\frac{1}{4}x + 2 = 0$

$+\frac{1}{4}x \quad +\frac{1}{4}x$

$4 \cdot 2 = \cancel{\frac{4}{1}} \cdot \frac{1}{4}x$

$8 = x$

Int (8,0)



11) Suppose f(x) = 3x – 6 and g(x) = - 2x + 4

a) Solve f(x) = 0

b) Solve f(x) > 0

c) Solve f(x) = g(x)

d) Solve f(x) < g(x)

11 [redacted] Suppose f(x) = $3x-6$ [redacted] and g(x) = $-2x+4$ [redacted]

a) Solve f(x) = 0

$$\begin{aligned} 3x-6 &= 0 \\ +6 \quad & \ \ +6 \\ \hline 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \end{aligned}$$

$$\boxed{x=2}$$

b) Solve f(x) > 0

$$\begin{aligned} 3x-6 &> 0 \\ +6 \quad & \ \ +6 \\ \hline 3x &> 6 \\ \frac{3x}{3} &> \frac{6}{3} \end{aligned}$$

$$\boxed{x>2}$$

c) Solve f(x) = g(x)

$$\begin{aligned} 3x-6 &= -2x+4 \\ +2x+6 \quad & \ \ +2x+6 \\ \hline 5x &= 10 \\ \frac{5x}{5} &= \frac{10}{5} \end{aligned}$$

$$\boxed{x=2}$$

d) Solve f(x) < g(x)

$$\begin{aligned} 3x-6 &< -2x+4 \\ +2x+6 \quad & \ \ +2x+6 \\ \hline 5x &< 10 \\ \frac{5x}{5} &< \frac{10}{5} \end{aligned}$$

$$\boxed{x<2}$$

13) Suppose f(x) = x-3 and g(x) = 2x + 4

a) Solve f(x) = 0

b) Solve f(x) > 0

c) Solve f(x) = g(x)

d) Solve f(x) < g(x)

13) Suppose f(x) = x-3 and g(x) = 2x+4

a) Solve f(x) = 0

$$\begin{array}{r} x-3=0 \\ +3\ +3 \\ \hline \end{array}$$

$\boxed{x=3}$

b) Solve f(x) > 0

$$\begin{array}{r} x-3>0 \\ +3\ +3 \\ \hline \end{array}$$

$\boxed{x>3}$

c) Solve f(x) = g(x)

$$\begin{array}{r} 1x-3=2x+4 \\ -1x-4\quad -1x-4 \\ \hline -7=x \end{array}$$

$\boxed{x=-7}$

d) Solve f(x) < g(x)

$$\begin{array}{r} 1x-3<2x+4 \\ -2x+3\ -2x+3 \\ \hline -1x<7 \\ x>-7 \end{array}$$

Switch Direction

$\boxed{x>-7}$

#15-20:

a) Create a scatter plot of the data.

b) Determine whether the given function is linear or nonlinear.

c) If the function is linear, determine the equation of the line. (Write your answer using function notation)

15)

x	y = f(x)
1	2
2	4
3	6
4	8
5	10
6	12

15b) Linear

15c) Pick Any 2 points, all will give Same Answer

(1,2)
(2,4)

$m = \frac{4-2}{2-1} = \frac{2}{1} = 2$

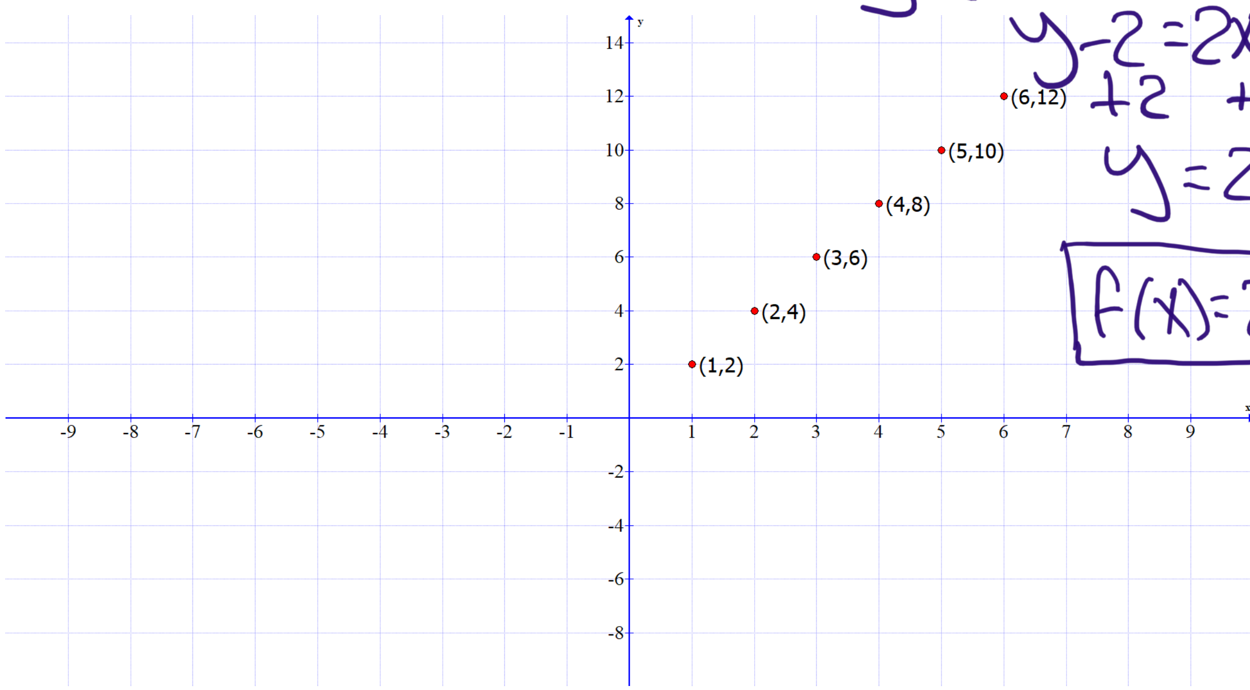
$m = 2$ Point (1,2)

$y-2 = 2(x-1)$

$y-2 = 2x-2$
$+2 \quad +2$

$y = 2x$

$\boxed{f(x) = 2x}$



#15-20:

a) Create a scatter plot of the data.

b) Determine whether the given function is linear or nonlinear.

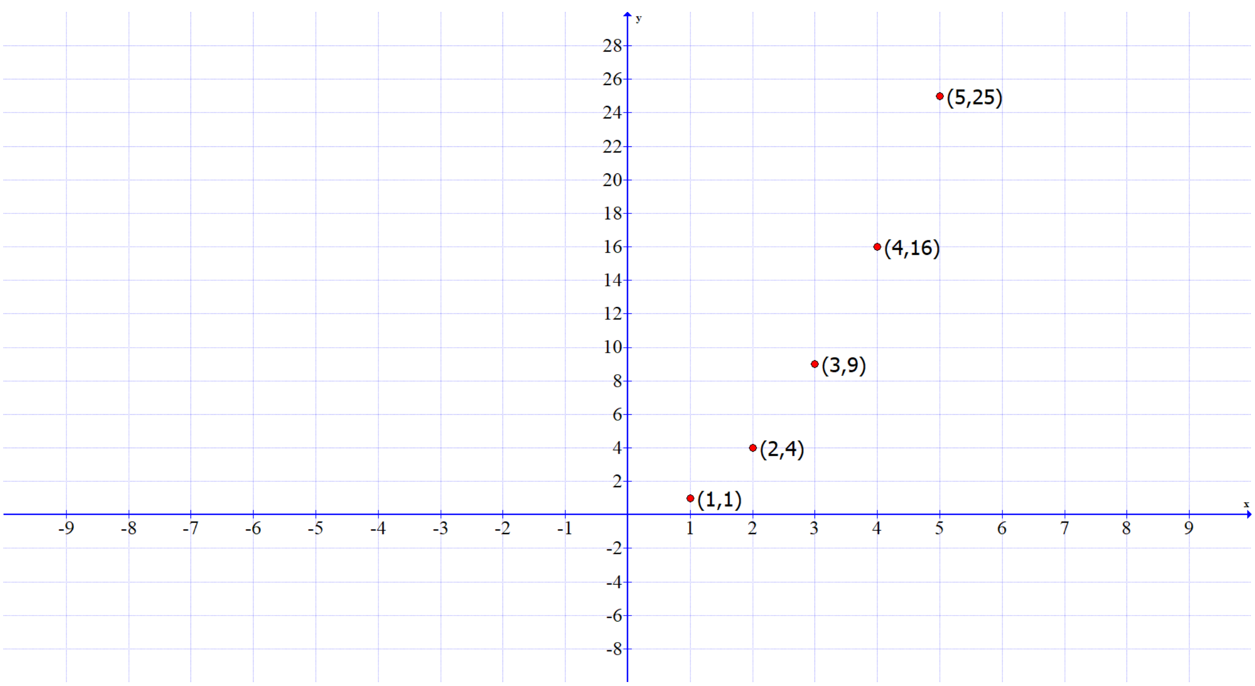
c) If the function is linear, determine the equation of the line. (Write your answer using function notation)

17)

x	y = f(x)
1	1
2	4
3	9
4	16
5	25

17b) data is not Linear

17c) Skip/ not required



#15-20:

a) Create a scatter plot of the data.

b) Determine whether the given function is linear or nonlinear.

c) If the function is linear, determine the equation of the line. (Write your answer using function notation)

19)

x	y = f(x)
1	20
2	15
3	10
4	5
5	0
6	-5
7	-10

19b) data is Linear

19c) (1,20) (2,15)

$$m = \frac{15-20}{2-1} = \frac{-5}{1} = -5$$

$m = -5$ point (1,20)

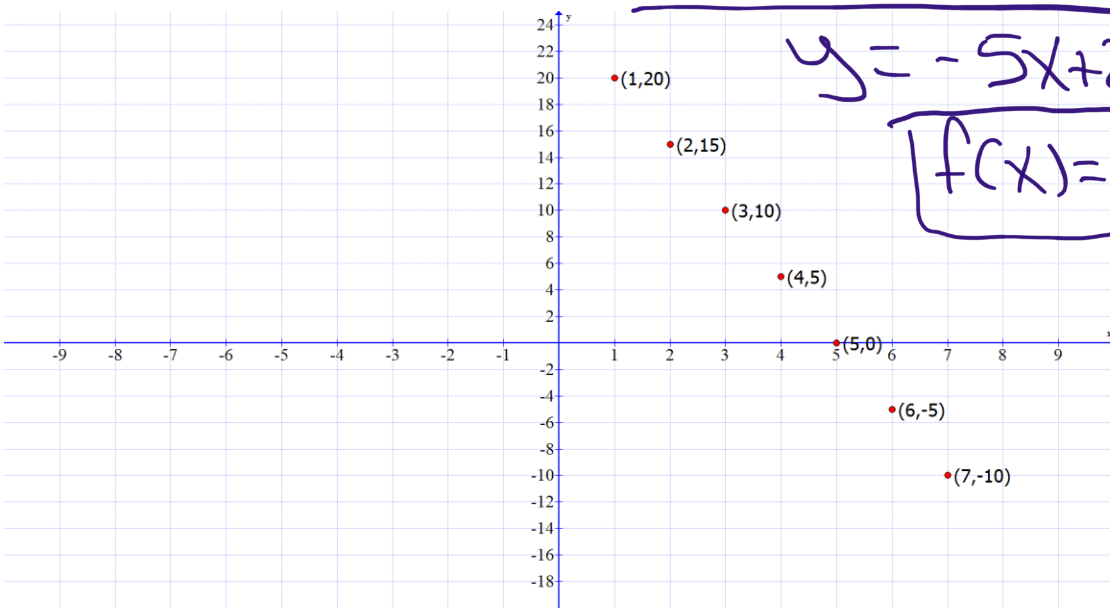
$$y - 20 = -5(x-1)$$

$$y - 20 = -5x + 5$$

$$+20 \qquad\qquad +20$$

$$y = -5x + 25$$

$$\boxed{f(x) = -5x + 25}$$



21) Suppose that the number of a units of a certain product that will be supplied (S) at price (p) (in dollars) is given by the equation:

$$S(p) = 2p - 10$$

Suppose that number of units of the same product that will be demanded (D) at price (p) (in dollars) is given by the equation:

$$D(p) = -2p + 20$$

a) How many units of the product will be supplied at a price of $8?

$S(8) = 2(8) - 10 = 16 - 10$ $\boxed{6 \text{ units}}$

b) How many units of the product will be demanded at a price of $8?

$D(8) = -2(8) + 20 = -16 + 20$ $\boxed{4 \text{ units}}$

c) At a price of $8 does the supply exceed demand, or does demand exceed ~~supply~~? supply

Supply exceeds demand

d) Find the equilibrium price. $7.50

e) How many units of the product will be supplied at the equilibrium price?

$S(7.50) = 2(7.50) - 10 = 15 - 10$ $\boxed{5 \text{ units}}$

f) How many units of the product will be demanded at the equilibrium price?

$D(7.50) = -2(7.50) + 20 = -15 + 20$ $\boxed{5 \text{ units}}$

21d) $2p - 10 = -2p + 20$

$+2p + 10 \quad +2p + 10$

$$\frac{4p}{4} = \frac{30}{4}$$

$p = 15/2$

Eq. Price $7.50

23) Suppose that the number of a units of a certain product that will be supplied (S) at price (p) (in dollars) is given by the equation:

$$S(p) = 5p - 40$$

Suppose that number of units of the same product that will be demanded (D) at price (p) (in dollars) is given by the equation:

$$D(p) = -3p + 40$$

a) How many units of the product will be supplied at a price of $9?

$S(9) = 5(9) - 40 = 45 - 40$ $\boxed{\text{5 units}}$

b) How many units of the product will be demanded at a price of $9?

$D(9) = -3(9) + 40 = -27 + 40$ $\boxed{\text{13 units}}$

c) At a price of $9 does the supply exceed demand, or does demand exceed ~~price~~? Supply

DEmand exceeds supply

d) Find the equilibrium price.

$10

e) How many units of the product will be supplied at the equilibrium price?

$S(10) = 5(10) - 40 = 50 - 40$ $\boxed{\text{10 units}}$

f) How many units of the product will be demanded at the equilibrium price?

$D(10) = -3(10) + 40 = -30 + 40 =$ $\boxed{\text{10 units}}$

23d) $5p - 40 = -3p + 40$

$+3p + 40 \quad +3p + 40$

$8p = 80$

$p = 10$

25) A company makes a single product. The monthly cost (C) to make x units of the product can be found using the cost equation:

$C(x) = 3x + 100$

The monthly revenue (R) earned from selling x units of the product can be found using the revenue equation:
$R(x) = 8x$

a) Find the cost of making 30 units of the product during a month.

$C(30) = 3(30) + 100 = 90 + 100 = \boxed{\$190}$

b) Find the monthly revenue earned by selling 30 units of the product.

$R(30) = 8(30) = 240$ $\boxed{\$240}$

c) Is there a profit or loss when 30 units of the product are produced and sold in a month?

Profit (Revenue exceeds cost)

d) What is the amount of the profit or loss?

$240 - 190$ $\boxed{\$50}$

e) Find the break-even quantity.

$$\begin{array}{rcl} 3x + 100 &=& 8x \\ -3x && -3x \\ \hline 100 &=& 5x \end{array}$$

$\boxed{20 \text{ units}}$

f) What is the monthly cost at the break-even quantity?

$C(20) = 3(20) + 100$ $\boxed{\$160}$

g) What is the monthly revenue at the break-even quantity?

$R(20) = 8(20)$ $\boxed{\$160}$

h) What is the monthly profit at the break-even quantity?

$160 - 160 = 0$ $\boxed{\$0}$

27) A company makes a single product. The monthly cost (C) to make x units of the product can be found using the cost equation:

$C(x) = 5x + 400$

The monthly revenue (R) earned from selling x units of the product can be found using the revenue equation:
$R(x) = 7x$

a) Find the cost of making 100 units of the product during a month.

$C(100) = 5(100) + 400$ $900

b) Find the monthly revenue earned by selling 100 units of the product.

$R(100) = 7(100)$ $700

c) Is there a profit or loss when 1000 units of the product are produced and sold in a month? loss

d) What is the amount of the profit or loss? $200 loss

$700 - 900 = -200$ OR $-200 profit

e) Find the break-even quantity.

$$\begin{array}{r} 5x + 400 = 7x \\ -5x \qquad\quad -5x \\ \hline 400 = 2x \end{array}$$

$\boxed{200 \text{ units}}$

f) What is the monthly cost at the break-even quantity?

$C(200) = 5(200) + 400$ $\boxed{\$1400}$

g) What is the monthly revenue at the break-even quantity?

$R(200) = 7(200)$ $\boxed{\$1400}$

h) What is the monthly profit at the break-even quantity?

$1400 - 1400 = 0$ $\boxed{\$0}$

29) U-Haul charges $25 per day plus 25 cents for each mile driven to rent a certain truck.

a) Create a linear function the models the cost of renting a truck for one day when "m" miles are driven.

$$C(m) = 0.25m + 25$$

b) What is the cost of renting the truck for one day if it is driven 100 miles?

$$C(100) = 0.25(100) + 25$$

$$\boxed{\$50}$$

c) Suppose the cost of renting a truck for 1 day is $75. How many miles were driven?

$$75 = 0.25m + 25$$

$$-25 \qquad\qquad -25$$

$$\frac{50}{0.25} = \frac{0.25m}{0.25}$$

$$200 = m$$

$$\boxed{200 \text{ miles}}$$

31) U-Haul charges $20 per day plus 50 cents for each mile driven to rent a certain truck.

a) Create a linear function the models the cost of renting a truck for one day when "m" miles are driven.

$$C(m) = 0.50m + 20$$

b) What is the cost of renting the truck for one day if it is driven 80 miles?

$$C(80) = 0.50(80) + 20 = \boxed{\$60}$$

c) Suppose the cost of renting a truck for 1 day is $35. How many miles were driven?

$$35 = 0.50m + 20$$
$$-20 \qquad\qquad -20$$
$$\frac{15}{0.50} = \frac{0.50m}{0.50}$$

$$\boxed{\text{30 miles}}$$